Basic Operational Amplifier Circuits
Comparators

A comparator is a specialized nonlinear op-amp circuit that compares two input voltages and produces an output state that indicates which one is greater. Comparators are designed to be fast and frequently have other capabilities to optimize the comparison function.

An example of a comparator application is shown. The circuit detects a power failure in order to take an action to save data. As long as the comparator senses $V_{in}$, the output will be a dc level.
Comparators

- Operational amplifiers are often used as comparators to compare the amplitude of one voltage with another.
- In this application, op-amps are used in the open-loop configuration.
- Due to high open-loop gain, an op-amp can detect very tiny differences at the input.
- The input voltage is applied to one terminal while a reference voltage on the other terminal.
- Comparators are much faster than op-amps.
- Op-amps can be used as comparators but comparators cannot be used as op-amps.
Zero-Level Detection

- Figure 1(a) shows an op-amp circuit to detect when a signal crosses zero. This is called a zero-level detector.

- Notice that inverting input is grounded to produce a zero level and the input signal is applied to the noninverting terminal.
Zero-Level Detection

- Because of high open-loop gain, small difference voltage between the inputs drives the op-amp into saturation.

- Figure 1(b) shows the result of a sinusoidal input voltage applied to the noninverting input of the zero-level detector.

- When the sine wave is positive, the output is maximum positive level. When the sine wave is negative, the output is maximum negative level.

- The change from positive to negative or negative to positive always occurs when the wave crosses zero.

- Can be used as a squaring circuit to produce a square wave from a sine wave.
Nonzero-Level Detection

- The zero-level detector of Figure 1 can be modified to detect positive and negative voltage levels by connecting a fixed reference voltage source at the inverting input as shown in Figure 2(a).

- A more practical arrangement is shown in Figure 13–2(b) using a voltage divider to set the reference voltage, \( V_{\text{REF}} \), as:

\[
V_{\text{REF}} = \frac{R_2}{R_1 + R_2} (+V)
\]
Nonzero-Level Detection

- The circuit in Figure 2(c) uses a zener diode to set the reference voltage \( V_{\text{REF}} = V_Z \).

- As long as the input signal \( V_{\text{in}} \) remains less then \( V_{\text{REF}} \), the output remains at the maximum negative level.

- When the input voltage crosses the reference voltage, the output goes to its maximum positive voltage as shown in Figure 2(d).
Example

The input signal in Figure 13–3(a) is applied to the comparator in Figure 13–3(b). Draw the output showing its proper relationship to the input signal. Assume the maximum output levels of the comparator are ±14 V.
The reference voltage is set by $R_1$ and $R_2$ as follows:

$$V_{REF} = \frac{R_2}{R_1 + R_2} (+V) = \frac{1.0 \text{ k}\Omega}{8.2 \text{ k}\Omega + 1.0 \text{ k}\Omega} (+15 \text{ V}) = 1.63 \text{ V}$$

As shown in Figure 13-4, each time the input exceeds $+1.63 \text{ V}$, the output voltage switches to its $+14 \text{ V}$ level, and each time the input goes below $+1.63 \text{ V}$, the output switches back to its $-14 \text{ V}$ level.
In practical situations, noise appears on the input signal. This noise voltage disturbs the input voltage as shown.

Noise can cause a comparator to erratically switch output states.
Consider a zero-level detector and a sinusoidal voltage input at the noninverting input of the comparator.

The input sine wave and the resulting output voltage are shown.

When the input voltage reaches zero, the disturbance due to noise may cause the input to fluctuate about the zero voltage value many times and thus producing an output that is not the desired one.
Effect of Input Noise on Comparator Operations

- Whenever the input signal hovers around the reference voltage, any small disturbance like noise will produce disturbed output.
- To reduce this noise effect, a technique called hysteresis is used.
- This requires the comparator to be used with positive feedback.
- The idea is to have 2 reference voltages. One reference is higher and the other is lower.
Comparator with Hysteresis

- The higher reference is for when the input signal goes from lower voltage to higher one and the lower reference is for when the input signal goes from higher to lower voltage.

- The two references are called upper trigger point (UTP) and lower trigger point (LTP).

- This two-level hysteresis is established with a positive feedback.

- The noninverting end is connected to a resistive voltage divider such that a portion of the output voltage is fed back to the input.

- The input signal is applied to the inverting input.

- The basic operation of the comparator with hysteresis is shown in next slide.
Comparator with Hysteresis

(b) When the output is at the maximum negative voltage and the input goes below LTP, the output switches back to the maximum positive voltage.

(a) When the output is at the maximum positive voltage and the input exceeds UTP, the output switches to the maximum negative voltage.

(c) Device triggers only once when UTP or LTP is reached; thus, there is immunity to noise that is riding on the input signal.
 Comparator with Hysteresis

- Assume the output voltage is at its positive maximum, $V_{out(\text{max})}$.
- The voltage fed back to the noninverting input is $V_{\text{UTP}}$ and is given as

$$V_{\text{UTP}} = \frac{R_2}{R_1 + R_2} (+V_{out(\text{max})})$$

- When $V_{\text{in}}$ exceeds $V_{\text{UTP}}$, the output voltage drops to its negative maximum, $-V_{out(\text{max})}$. The voltage fed back to the noninverting input is $V_{\text{LTP}}$ and is given as

$$V_{\text{LTP}} = \frac{R_2}{R_1 + R_2} (-V_{out(max)})$$
Comparator with Hysteresis

- The input voltage must now fall below $V_{TLP}$ before the device will switch from maximum negative voltage to maximum positive voltage.

- This means that a small amount of noise voltage has no effect on the output.

- The comparator with built-in hysteresis is sometimes known as a Schmitt trigger.

- The amount of hysteresis is defined as the difference of the two trigger levels.

$$V_{HYS} = V_{UTP} - V_{LTP}$$
A comparator with hysteresis is also called a Schmitt trigger. The trigger points are found by applying the voltage-divider rule:

\[
V_{\text{UTP}} = \frac{R_2}{R_1 + R_2} (+V_{\text{out(max)}}) \quad \text{and} \quad V_{\text{LTP}} = \frac{R_2}{R_1 + R_2} (-V_{\text{out(max)}})
\]

**Example:**

What are the trigger points for the circuit if the maximum output is ±13 V?

**Solution:**

\[
V_{\text{UTP}} = \frac{R_2}{R_1 + R_2} (+V_{\text{out(max)}}) = \frac{10 \, \text{k}\Omega}{47 \, \text{k}\Omega + 10 \, \text{k}\Omega} (+13 \, \text{V}) = 2.28 \, \text{V}
\]

By symmetry, the lower trigger point = −2.28 V.
The output swing of a zero-crossing detector may be too large in some applications.

In some applications, it is necessary to limit the output voltage levels of comparator to a value less than provided by the saturated op-amp.

We can bound the output by using a zener diode – limit the output voltage to the zener voltage in one direction.
The anode of the zener is connected to the inverting input. When output voltage reaches positive value equal to the zener voltage, it limits at that value. At negative output, zener acts as a regular diode and becomes forward biased at 0.7V and limits the negative output voltage to this value.
The cathode of the zener is connected to the inverting input.

The output voltage limits in the opposite direction.
Two zener diodes arranged – limit the output voltage to the zener voltage plus forward biased 0.7V (positively and negatively).
Example

Determine the output voltage waveform for Figure 13–13.

Solution
This comparator has both hysteresis and zener bounding. The voltage across $D_1$ and $D_2$ in either direction is $4.7 \, \text{V} + 0.7 \, \text{V} = 5.4 \, \text{V}$. This is because one zener is always forward-biased with a drop of 0.7 V when the other one is in breakdown.
Example

The voltage at the inverting (−) op-amp input is $V_{out} \pm 5.4$ V. Since the differential voltage is negligible, the voltage at the noninverting (+) op-amp input is also approximately $V_{out} \pm 5.4$ V. Thus,

$$V_{R1} = V_{out} - (V_{out} \pm 5.4 \text{ V}) = \pm 5.4 \text{ V}$$

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{\pm 5.4 \text{ V}}{100 \text{ k}\Omega} = \pm 54 \mu\text{A}$$

Since the noninverting input current is negligible,

$$I_{R2} = I_{R1} = \pm 54 \mu\text{A}$$

$$V_{R2} = R_2I_{R2} = (47 \text{ k}\Omega)(\pm 54 \mu\text{A}) = \pm 2.54 \text{ V}$$

$$V_{out} = V_{R1} + V_{R2} = \pm 5.4 \text{ V} \pm 2.54 \text{ V} = \pm 7.94 \text{ V}$$

The upper trigger point (UTP) and the lower trigger point (LTP) are as follows:

$$V_{UTP} = \left( \frac{R_2}{R_1 + R_2} \right)(+V_{out}) = \left( \frac{47 \text{ k}\Omega}{147 \text{ k}\Omega} \right)(+7.94 \text{ V}) = +2.54 \text{ V}$$

$$V_{LTP} = \left( \frac{R_2}{R_1 + R_2} \right)(-V_{out}) = \left( \frac{47 \text{ k}\Omega}{147 \text{ k}\Omega} \right)(-7.94 \text{ V}) = -2.54 \text{ V}$$
The output waveform for the given input voltage is shown in Figure 13–14.
Comparator Applications

Over Temperature Sensing Circuit
Over Temperature Sensing Circuit

- Used to determine when the temperature reaches a certain critical value. The circuit consists of a Wheatstone bridge with the op-amp used to detect when the bridge is balanced. One leg of the bridge contains a thermistor (R₁), which is a temperature-sensing resistor with a negative temperature coefficient (its resistance decreases as temperature increases).

- The potentiometer (R₂) is set at a value equal to the resistance of the thermistor at the critical temperature.

- At normal temperatures (below critical), R₁ is greater than R₂, thus creating an unbalanced condition that drives the op-amp to its low saturated output level and keeps transistor Q₁ off.
Over Temperature Sensing Circuit

- As the temperature increases, the resistance of the thermistor decreases.
- When the temperature reaches the critical value, $R_1$ becomes equal to $R_2$, and the bridge becomes balanced (since $R_3 = R_4$).
- At this point the op-amp switches to its high saturated output level, turning Q1 on. This energizes the relay, which can be used to activate an alarm or initiate an appropriate response to the over-temperature condition.
A/D Conversion

Simultaneous or flash analog-to-digital converters use $2^n - 1$ comparators to convert an analog input to a digital value for processing. Flash ADCs are a series of comparators, each with a slightly different reference voltage. The priority encoder produces an output equal to the highest value input.

In IC flash converters, the priority encoder usually includes a latch that holds the converter data constant for a period of time after the conversion.
Determine the binary number sequence of the three-digit simultaneous ADC in Figure 13–16 for the input signal in Figure 13–17 and the sampling pulses (encoder enable) shown. Draw the resulting digital output waveforms.
The resulting binary output sequence is listed as follows and is shown in the waveform diagram of Figure 13–18 in relation to the sampling pulses.

011, 101, 110, 110, 100, 001, 000, 001, 010, 101, 110, 111
A summing amplifier has two or more inputs and its output is the negative algebraic sum of its input voltages.

A two-input summing amplifier is shown. Both the input voltages are applied to the inverting input.

The output voltage for the amplifier can be written as

\[ V_{OUT} = -(I_1 + I_2)R_f = -(\frac{V_{IN1}}{R_1} + \frac{V_{IN2}}{R_2})R_f \]

If all three of the resistors are equal, then:

\[ V_{OUT} = -(V_{IN1} + V_{IN2}) \]
The previous equation shows that the output voltage has the same magnitude as the sum of the two input voltages but with negative sign.

A general expression for a unity-gain summing amplifier with \( n \) inputs as shown in Figure where all resistances are equal is given by:

\[
V_{OUT} = -(V_{IN1} + V_{IN2} + V_{IN3} + \cdots + V_{INn})
\]
Example:

What is $V_{OUT}$ if the input voltages are +5.0 V, −3.5 V and +4.2 V and all resistors = 10 kΩ?

Solution:

$$V_{OUT} = -(V_{IN1} + V_{IN2} + V_{IN3})$$

$$= -(+5.0 \text{ V} - 3.5 \text{ V} + 4.2 \text{ V})$$

$$= -5.7 \text{ V}$$
Example:

Determine the output voltage for the summing amplifier

Solution:

\[ V_{\text{OUT}} = -\frac{R_f}{R} (V_{\text{IN1}} + V_{\text{IN2}}) \]

\[ = -\frac{10\,\text{k}\Omega}{1.0\,\text{k}\Omega} (0.2\,\text{V} + 0.5\,\text{V}) \]

\[ = -5.7\,\text{V} \]
An **averaging amplifier** is basically a summing amplifier with the gain set to $R_f/R = 1/n$ ($n$ is the number of inputs). The output is the negative average of the inputs.

**Example:**

What is $V_{OUT}$ if the input voltages are $+5.0$ V, $-3.5$ V and $+4.2$ V? Assume $R_1 = R_2 = R_3 = 10 \text{ k}\Omega$ and $R_f = 3.3 \text{ k}\Omega$?

**Solution:**

\[
V_{OUT} = -\frac{1}{3}(V_{IN1} + V_{IN2} + V_{IN3})
\]
\[
= -\frac{1}{3}(+5.0 \text{ V} - 3.5 \text{ V} + 4.2 \text{ V})
\]
\[
= -1.9 \text{ V}
\]
A **scaling adder** has two or more inputs with each input having a different gain. The output represents the negative *scaled* sum of the inputs.

**Example:**

Assume you need to sum the inputs from three microphones. The first two microphones require a gain of $-2$, but the third microphone requires a gain of $-3$. What are the values of the input $R$’s if $R_f = 10 \, \text{k}\Omega$?

**Solution:**

\[
R_1 = R_2 = -\frac{R_f}{A_{v1}} = -\frac{10 \, \text{k}\Omega}{-2} = 5.0 \, \text{k}\Omega
\]

\[
R_3 = -\frac{R_f}{A_{v3}} = -\frac{10 \, \text{k}\Omega}{-3} = 3.3 \, \text{k}\Omega
\]
An application of a **scaling adder** is the D/A converter circuit shown here. The resistors are inversely proportional to the binary column weights. Because of the precision required of resistors, the method is useful only for small DACs.
Determine the output voltage of the DAC in Figure 13–27(a). The sequence of four-digit binary codes represented by the waveforms in Figure 13–27(b) are applied to the inputs. A high level is a binary 1, and a low level is a binary 0. The least significant binary digit is $D_0$.

\[ V_{OUT(D_0)} = -\frac{10}{200} \times 5 = -0.25 \, V \]

\[ V_{OUT(D_1)} = -0.5 \, V, \quad V_{OUT(D_2)} = -1 \, V, \quad V_{OUT(D_3)} = -2 \, V \]
A more widely used method for D/A conversion is the $R/2R$ ladder. The gain for $D_3$ is $-1$. Each successive input has a gain that is half of previous one. The output represents a weighted sum of all of the inputs (similar to the scaling adder).

$$R_f = 2R$$
(a) Equivalent circuit for $D_3 = 1, D_2 = 0, D_1 = 0, D_0 = 0$

(b) Equivalent circuit for $D_3 = 0, D_2 = 1, D_1 = 0, D_0 = 0$
Difference Amplifier

This circuit is also called a differential amplifier, since it amplifies the difference between the input signals.

\[ v_o = v_- i_2 R_2 = v_- i_1 R_2 \]
\[ = v_- \frac{R_2}{R_1} (v_1 - v_-) = \left( \frac{R + R_2}{R_1} \right) v_- \frac{R_2}{R_1} v_1 \]

Also, \[ v_+ = \frac{R_2}{R_1 + R_2} v_2 \]

Since \[ v_- = v_+ \]
\[ v_o = -\frac{R_2}{R_1} (v_1 - v_2) \]

For \( R_2 = R_1 \)
\[ v_o = -(v_1 - v_2) \]

- This circuit is also called a differential amplifier, since it amplifies the difference between the input signals.
- \( R_{in2} \) is series combination of \( R_1 \) and \( R_2 \) because \( i_+ \) is zero.
- For \( v_2 = 0 \), \( R_{in1} = R_f \), as the circuit reduces to an inverting amplifier.
- For general case, \( i_1 \) is a function of both \( v_1 \) and \( v_2 \).
- Differential input resistance: $R_{id} = 2R_1$
- Large $R_1$ can be used to increase $R_{id}$
- $R_2$ becomes impractically large to maintain required gain.
**Integrators and Differentiators**

**Inverting configuration with general impedance**

$R_1$ and $R_2$ in inverting configuration can be replaced by $Z_1(s)$ and $Z_2(s)$.

The closed-loop transfer function: $V_o(s) / V_i(s) = -Z_2(s) / Z_1(s)$

The transmission magnitude and phase for a sinusoid input can be evaluated by replacing $s$ with $j\omega$. 
The **ideal integrator** is an inverting amplifier that has a capacitor in the feedback path. The output voltage is proportional to the negative integral of the input voltage.

From the Figure we have

\[ I_{in} = \frac{V_{in}}{R_i} = I_C = C \frac{dV_C}{dt} \]

\[ V_{out} = -\frac{1}{R_i C} \int_0^t V_{in}(\tau)d\tau \]

When a constant positive input voltage in the form of a step or pulse is applied, the output is a ramp that decreases negatively until the op-amp saturates at its maximum negative level.
The Ideal Integrator

The Output Voltage is the same as the voltage on the negative side of the capacitor. The rate at which the capacitor charges, and therefore the slope of the output ramp is:

\[
\frac{\Delta V_{out}}{\Delta t} = -\frac{V_{in}}{R_{i}C}
\]
The Ideal Integrator

Frequency domain analysis

\[ \frac{V_o}{V_i} = -\frac{1}{j\omega R_i C} \]

The capacitor behaves as an open-circuit at dc \((\omega = 0)\). This means that open-loop configuration at dc (infinite gain). Any tiny dc in the input could result in output saturation.
The ideal integrator uses a capacitor in the feedback which is open to DC.

This implies that for DC voltage the capacitor becomes open and the op-amp becomes open-loop.

Op-amp integrating circuits must have extremely low dc offset and bias currents, because small errors are equivalent to a dc input. The ideal integrator tends to accumulate these errors, which moves the output toward saturation.

The practical integrator overcomes these errors— the simplest method is to add a relatively large feedback resistor.
Practical Integrator

In order to prevent integrator saturation due to infinite dc gain, parallel feedback resistance is included.

\[ \frac{V_o}{V_i} = -\frac{1}{R_i/R_f + j\omega R_i C} \]

Closed-loop gain = \(-1/(j\omega R_i C + R_i/R_f)\)

Closed-loop gain at dc = \(-R_i/R_f\)

Closed-loop gain at high frequency = \(-1/j\omega R_i C\)

The integrator characteristics is no longer ideal

Large resistance \(R_f\) should be used for the feedback
Sketch the output wave for the shown input

\[ V_{in} = \begin{cases} +2.0 \text{ V} & \text{for } 0 \leq t < 0.5 \\ 0 \text{ V} & \text{for } 0.5 \leq t < 1.0 \\ -2.0 \text{ V} & \text{for } 1.0 \leq t < 2.0 \end{cases} \]

\[ t (\text{ms}) \]

\[ V_{out} = \begin{cases} +1.0 \text{ V} & \text{for } 0 \leq t < 1.0 \\ 0 \text{ V} & \text{for } 1.0 \leq t < 1.5 \\ -1.0 \text{ V} & \text{for } 1.5 \leq t < 2.0 \end{cases} \]

\[ t (\text{ms}) \]

Solution:

\[ \Delta V_{out} \over \Delta t = -\frac{V_{in}}{R_C} = \frac{2 \text{ V}}{(10 \text{ k}\Omega)(0.1 \mu\text{F})} = 2 \text{ V/ms} \]
Example

Find the output produced by an integrator in response to an input pulse of 1-V height and 1-ms width. Let $R = 10 \ \text{k}\Omega$ and $C = 10 \ \text{nF}$.

If the integrator capacitor is shunted by a 1-M$\Omega$ resistor, how will the response be modified? The op amp is specified to saturate at ±13 V.

**Solution:**

\[
\frac{V_{\text{in}}}{RC} = -\frac{1}{(10\text{k}\Omega)(10\text{nF})} = 10 \ \text{V}/\text{ms}
\]
With resistor connected across C, the 1-V pulse will provide a constant current \( I = 0.1 \text{ mA} \). This current is supplied to an STC network composed of \( R_F \) in parallel with \( C \). The output will be an exponential heading toward \(-100 \text{ V}\) with a time constant of \( R_F C = 10 \text{ ms} \)

\[
v_O(t) = -100(1 - e^{-t/10}), \quad 0 \leq t \leq 1 \text{ ms}
\]

\[
v_O(1 \text{ ms}) = -100(1 - e^{-1/10}) = -9.5 \text{ V}
\]

For \( t > 1 \text{ ms} \), the capacitor discharges through \( R_F \) toward 0 V with time-constant of 10 ms. Op Amp saturation has no effect on the operation of this circuit.
The Ideal Differentiator

The capacitor is the input element, and the resistor is the feedback element. A differentiator produces an output that is proportional to the rate of change of the input voltage.

From the Figure we have

\[ I_{in} = C \frac{dV_{in}}{dt} = I_R = -\frac{V_{out}}{R_f} \]

\[ V_{out} = -R_f C \frac{dV_{in}}{dt} \]

Apply a positive ramp voltage to the input, the output is constant equals \(-R_f C\) multiplied by the slope of the ramp.
When input is a positive-going ramp, the output is negative (capacitor is charging)

When input is a negative-going ramp, the output is positive (capacitor is discharging) – current in the opposite direction
Determine the output voltage of the ideal op-amp differentiator in Figure 13–40 for the triangular-wave input shown.
The small reactance of $C$ at high frequencies means an ideal differentiator circuit has very high gain for high-frequency noise. To compensate for this, a small series resistor is often added to the input. This **practical differentiator** has reduced high frequency gain and is less prone to noise.
**Hysteresis** Characteristics of a circuit in which two different trigger levels produce an offset or lag in the switching action.

**Schmitt trigger** A comparator with built-in hysteresis.

**Bounding** The process of limiting the output range of an amplifier or other circuit.

**Integrator** A circuit that produces an output that approximates the area under the curve of the input function.

**Differentiator** A circuit that produces an output that approximates the instantaneous rate of change of the input function.
1. The signal that you would expect at the output of the comparator (red arrow) is a

a. series of alternating positive and negative triggers
b. sine wave
c. square wave
d. dc level
2. Hysteresis is incorporated in a comparator by adding
   a. a capacitor in series with the input
   b. capacitors from the power supply to ground
   c. a small resistor in series with the input
   d. positive feedback
3. To find the trigger points for a Schmitt trigger, you can
   a. divide the saturation voltage by two
   b. apply Kirchhoff’s Voltage Law
   c. apply the voltage-divider rule
   d. calculate the rate of change of the input
4. A comparator output can be limited (bounded) by
   a. reversing the power supply voltages
   b. putting a zener diode in a feedback path
   c. decreasing the input resistance
   d. connecting the inverting input to ground
5. Assume all resistors in the circuit shown here have the same value. The circuit is a

a. summing amplifier
b. averaging amplifier
c. scaling adder
d. none of the above
6. Assume all resistors in the circuit shown here have different values. The circuit is a

a. summing amplifier
b. averaging amplifier
c. scaling adder
d. none of the above
7. The circuit shown is a
a. A/D converter
b. $R/2R$ ladder
c. both of the above
d. none of the above
8. A practical integrator has a feedback resistor in parallel with $C$. The purpose of this resistor is to

a. avoid noise

b. increase the gain

c. both of the above

d. none of the above
9. A certain circuit has the input and output signals shown. The circuit is

a. a differentiator
b. an integrator
c. a scaling amplifier
d. none of the above
10. A differentiator circuit produces an output that is proportional to the negative of the

a. sum of the inputs
b. rate of change of the input
c. area under the curve of the input
d. none of the above
Answers:

1. c  6. c
2. d  7. b
3. c  8. d
4. b  9. a
5. a  10. b